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Predictions of a Particle Model

Stanley Gudder[1]

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Previous papers began the development of a finite model for particles. Here various predictions of the model are derived. It is shown that the the model predicts a finite number of leptons and basic hadron isospin multiplets (127 to be precise). It predicts a different generation behavior for leptons and quarks than the standard model. A mass formula considered previously is used to predict masses of leptons and mesons that have not yet been observed.

1. PARTICLE GRAPHS

In a model for elementary particles presented in Gudder (1988, and to appear), a basic role was played by the symmetric group S_3. This group has six elements and it was used to describe the physical symmetries of the color set

$$S=\{r, y, b, \bar{r}, \bar{y}, \bar{b}\}$$

The Cayley graph corresponding to S_3 is illustrated in Figure 1.

In the Cayley graph, if we identify the vertex pairs $(r, \bar{r})$, $(y, \bar{y})$, $(b, \bar{b})$, we obtain a new graph B. If we identify the vertex triples (r, y, b), $(\bar{r}, \bar{y}, \bar{b})$, we obtain a graph M. Finally, if we identify all of the vertices, we obtain a graph L. These graphs are illustrated in Figure 2.

The graphs B, M, and L correspond to baryons, mesons, and leptons, respectively. We interpret the vertices as quarklike constituents and the edges represent strong (or color) interactions. More precisely, the edges represent possible interaction paths for the force mediating gluons. The vertex pair identification in B means that each of three vertices possesses one of the three colors r, y, b in the case of a particle and one of the three anticolors $\bar{r}$, $\bar{y}$, $\bar{b}$ in the case of an antiparticle. The triplet identification in

[1]Department of Mathematics and Computer Science, University of Denver, Denver, Colorado 80208.

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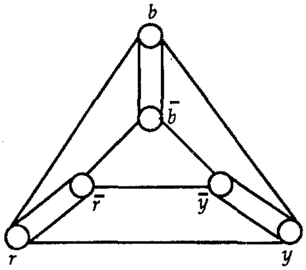

Fig. 1. Cayley graph.

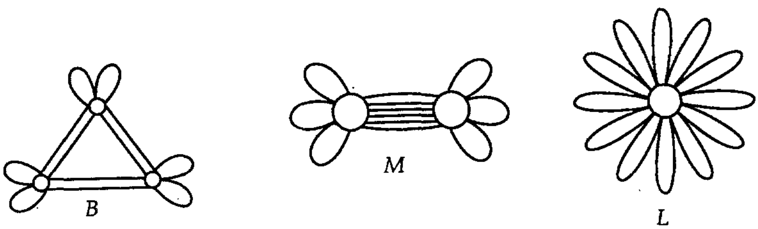

Fig. 2. The graphs B, M and L.

M means that one of the vertices possesses a color and the other an anticolor. The complete identification in L results in a single vertex having no color.

It was shown in Gudder (to appear) that in order to describe decay processes of elementary particles, edges can "migrate" from one vertex to another. We define a *migration* of B to be B itself or B altered by a migration of one or more edges. A migration of M is defined similarly. Figure 3 illustrates some migrations of B and M.

We call subgraphs of L, *lepton graphs*; subgraphs of migrations of M, *meson graphs*; and subgraphs of migrations of B, *baryon graphs*. In essence, if a graph has 12 or fewer edges with one, two, or three vertices, then it is a lepton, meson, or baryon graph, respectively. To describe a particle completely, we introduce the helicity and charge of a vertex. Each vertex has a (z component of) spin $\pm 1/2$ (up, down). Moreover, the vertex of a

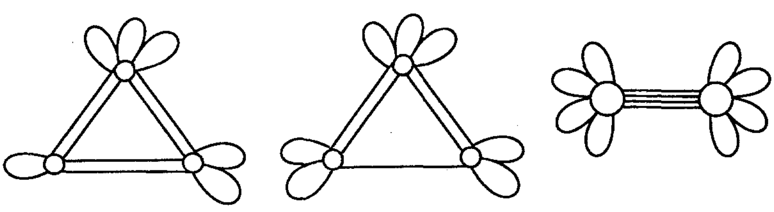

Fig. 3. Migrations of B and M.

lepton graph has (electric) charge 0 or ±1, while each vertex of a meson or baryon graph has charge ±1/3 or ±2/3. We now postulate the following charge rules for our graphs.

(C1) The sum of the charges is integral.

(C2) For a lepton graph, if the vertex has an odd number of loops, then its charge is ±1 and if the vertex has an even nonzero number of loops, then its charge is 0. For a meson or baryon graph, if a vertex has an odd number of loops, then its charge is ±1/3 and if a vertex has an even nonzero number of loops, then its charge is ±2/3.

Of course, C1 automatically holds for any lepton graph. If a lepton graph satisfies C2, it is called an *admissible lepton graph.* The admissible lepton graphs are illustrated in Figure 4. The first six are the known leptons, while the last eight form predicted new generations that we call τ^1, τ^2, τ^3, τ^4 together with their neutrinos. The numbers labeling the vertices designate the number of loops on that vertex. An unlabeled vertex has no loops. For simplicity, we do not label the charges.

We next postulate the following meson spin rules. Motivation for these rules are given in Gudder (1988, and to appear).

(M1) The two vertices have opposite spin if and only if they are joined by one or two edges. They have the same spin if and only if they are joined by three edges.

(M2) For opposite spin, if both vertices have at least one loop, then they are joined by two edges and if exactly one vertex has at least one loop, then they are joined by one edge.

A meson graph that obeys C1, C2, M1, M2 is called an *admissible meson graph.* We postulate that there is a one-to-one correspondence between meson isospin multiplets and admissible meson graphs. Figure 5 illustrates this correspondence by exhibiting all admissible meson graphs. In this figure an open circle represents a vertex with spin 1/2 and a filled circle represents a vertex with spin −1/2. We do not label the charges of vertices; if this is done, the individual members of a multiplet are obtained. The mesons π, K, η, ρ, K^* as well as the first few K_j, K_j^*, η_{jk}, and η_{jk}^* have been observed. Thus, the model predicts a total of 58 basic meson isospin multiplets [excited meson states can also be described (Gudder, 1988) but we shall not consider these here]. Notice that vertices with no loops correspond to down and up quarks, while vertices with 1, 2, 3, and

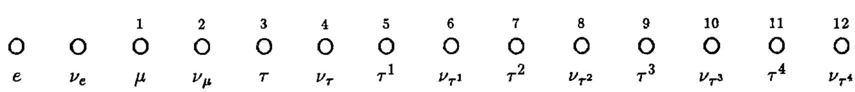

Fig. 4. Admissible lepton graphs.

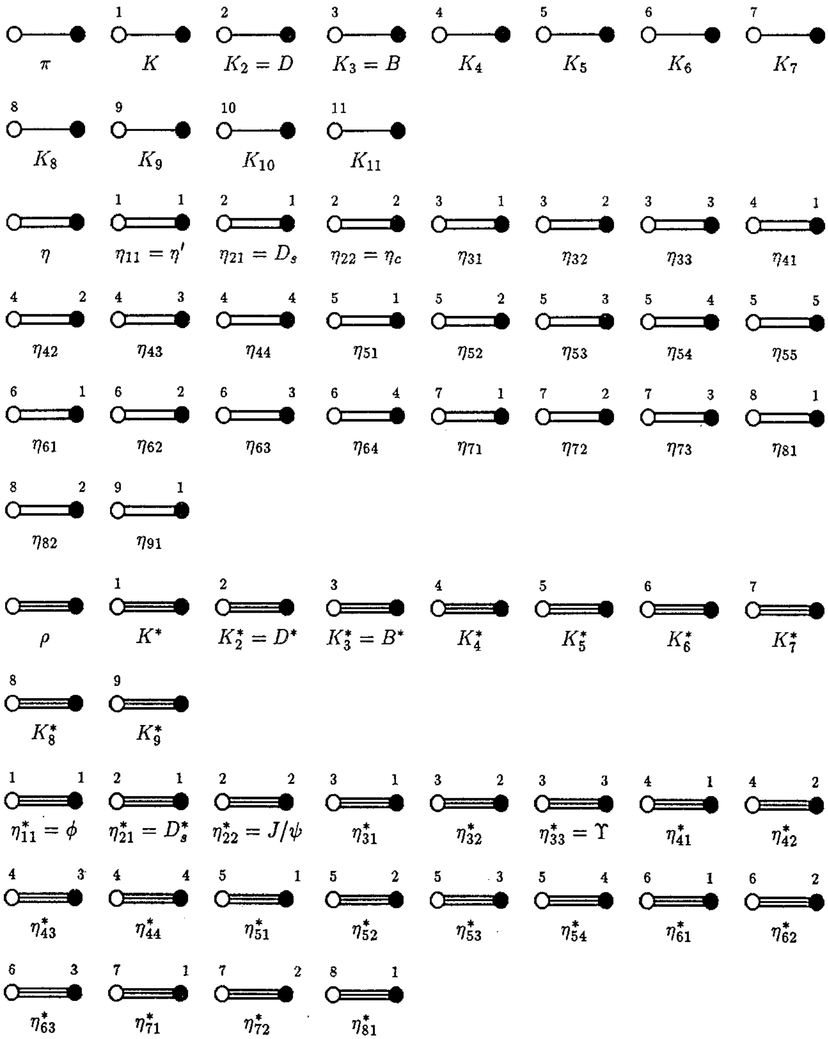

Fig. 5. Admissible meson graphs.

4 loops correspond to strange, charmed, bottom, and top quarks, respectively. Thus, there is no need to postulate various quark flavors in this model; these merely correspond to vertices with various numbers of loops. If the spins of the vertices are reversed, the particles (and their graphs) are considered to be the same. Finally, we postulate baryon spin rules. Again, these rules are motivated in Gudder (1988, and to appear).

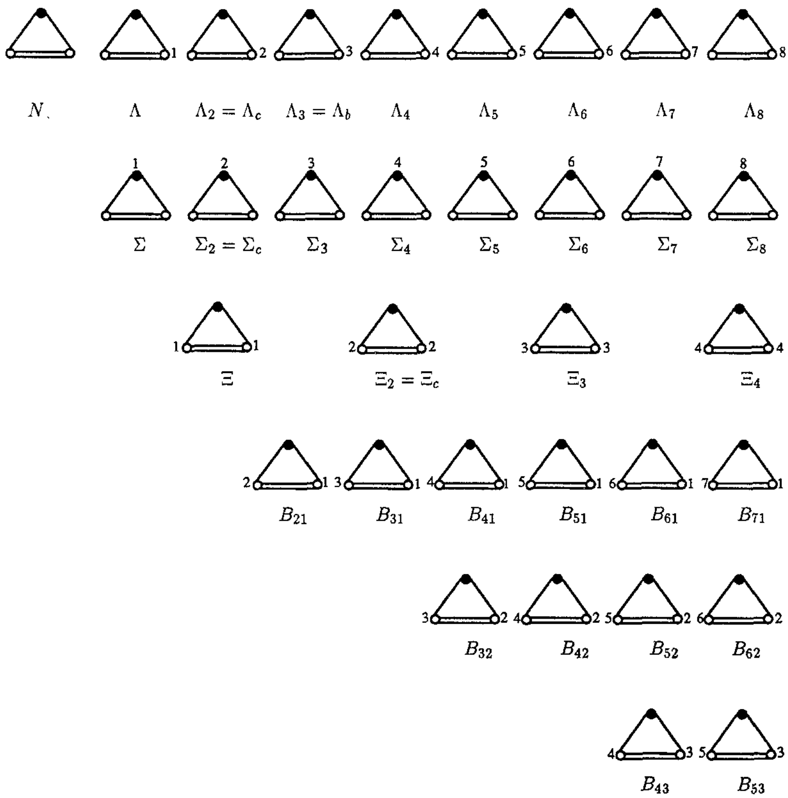

Fig. 6. Admissible baryon graphs, spin 1/2.

(B1) Two vertices have opposite spin if and only if they are joined by one edge and they have the same spin if and only if they are joined by two edges.

(B2) If two vertices have the same charge and the same number of loops, they have the same spin. If two vertices have different nonzero number of loops, they have the same spin.

A baryon graph that obeys C1, C2, B1, B2 is called an *admissible baryon graph.* We again postulate that there is a one-to-one correspondence by exhibiting all admissible baryon graphs (Figs. 6 and 7). Thus, the model predicts 55 basic baryon isospin multiplets.

2. PARTICLE MASSES

A partial derivation for mass formulas was given in Gudder (1988). The values given by these formulas agreed with the mass values of known

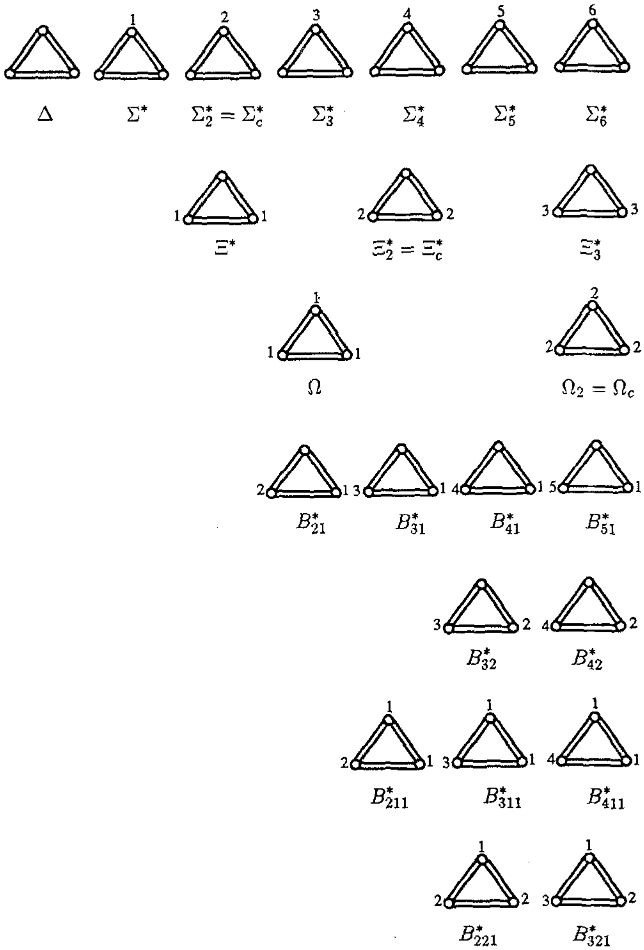

Fig. 7. Admissible baryon graphs, spin 3/2.

particles to within 1%. We now apply these formulas to predict the masses for some of the new particles derived in Section 1. In the present work we concentrate on the massive leptons and the mesons.

We postulate that the mass of a basic particle is the sum of two terms. The first term is the total mass of its vertices and the second term is the total energy of its gluons (which are assumed massless). We first consider the vertex mass. Let G be the graph of a basic particle and let u be a vertex of G. We denote by l the number of loops on u, by l' the number of loops on vertices of G other than u, and by s the (absolute) total spin of the particle. Let $\theta(x)=0$ if $x \leq 0$, $\theta(x)=1$ if $x>0$, and let d be the number of edges incident to u. In mass units for which π radians equals 70 MeV, we postulate that the mass of a vertex u is given as follows. For leptons

$$m_u = \pi\left[\tfrac{1}{2}+\tfrac{1}{3}l(l-1)\right]$$

For basic mesons

$$m_u = \pi\left[\tfrac{1}{2}+\max(f(l),\, \theta(d-1)g(l))\right]$$

where

$$f(l) = l(l-1)\left|\frac{4}{3}-(-1)^l(l-2)-3(l-3)\left[\frac{s}{2}-\frac{1}{5}(l-2)(l-8)\right]\right|$$

$$g(l) = \frac{8}{3}\,l'^2(l'-1)$$

The lepton formula and the first three terms of the meson formula were derived in Gudder (1988). We have added a new term which does not affect the work in Gudder (1988) since we only considered $l \leq 3$ there.

We also have a spin-loop interaction term in our meson mass formula. If l_1 denotes the total number of loops, then the total mass of the vertices in a basic meson is

$$M_v = \sum m_u - 2\pi s(l_1-2)\left[\frac{l_1}{3}-(l_1-3)\theta(l_1-1)\right]$$

For massive leptons (other than the electron, which we do not consider here) the formula for M_v is merely $M_v = m_u$.

We next consider gluon energies. It should first be understood that we use the name gluon to conform with the usual terminology, but it might be preferable to choose a different name such as "graphon." In the present model, the gluons (or graphons) mediate both the strong and weak interactions and they are even present in leptons. There is no need for introducing weak gauge bosons in this context. We assume that gluons perform a quantum random walk along the edges of a particle graph. Roughly speaking, when a gluon moves along an edge joining two different vertices, it mediates the strong force and when it moves along a loop, it mediates the weak force. The number of loops on a vertex, which determines the flavor of that vertex, affects the total gluon energy.

As described in Gudder (1988), the gluon random walk is dictated by an absorption–emission matrix T. The matrix T is the direct sum of matrices of the form

$$N_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

and

$$M_n = \frac{1}{\sqrt{n}} [e^{i\pi(j-k)^2/n}], \qquad n = 2, 3, \ldots, \quad 0 \leq j, k \leq n-1$$

For example, the tauon τ has three loops whose motions are assumed to be independent and the resulting T is given by the 6×6 matrix

$$T(\tau) = N_2 \oplus N_2 \oplus N_2$$

The nucleon N has two vertices of degree 3 and one vertex of degree 2. The resulting $T(N)$ is given by the 8×8 matrix

$$T(N) = M_3 \oplus M_3 \oplus M_2$$

Since the dynamics of a gluon is governed by a unitary matrix T, we conclude that the gluon energy values are related to the eigenvalues of T. The precise relationship between these two values is described in Gudder (1988). Denoting the total gluon energy of a particle by E_g, the mass of the particle is given by

$$m = M_v + E_g.$$

Using this equation, the predicted masses of the massive leptons (except the electron) and some of the mesons are summarized in Tables I and II. Since the eigenvalue calculations for the most massive mesons are quite tedious, we have only estimated them. This does not affect the result significantly, since the vertex mass terms dominate. Notice how close the masses of the predicted K_6^* and K_7^* mesons are to the weak gauge bosons W and Z, respectively. We conjecture that W and Z are actually these mesons. The experimental mass values are taken from M. Anguilar-Benitez, *et al.* (1988).

Table I. Lepton Masses

Lepton	M_v/π	E_g/π	Mass (GeV) $M_v + E_g$	Experimental value
μ	0.5	1	0.105	0.1056
τ	2.5	23	1.785	1.7845
τ^1	7.166	125	9.252	—
τ^2	14.5	371	26.985	—
τ^3	24.5	825	59.465	—
τ^4	37.17	1551	111.172	—

Table II. Meson Masses

Lepton	M_v/π	E_g/π	Mass (GeV) $M_v + E_g$	Experimental value
π	1	1	0.140	0.13957
K	1	6	0.490	0.4937
η	1	6.75	0.5425	0.5488
ρ	1	10	0.770	0.770
K^*	1.666	11.066	0.891	0.892
η_{11}	1	12.75	0.9625	0.9575
η_{11}^*	1	13.5	1.015	1.0194
K_2	3.666	23	1.867	1.8693
K_2^*	17.333	11.38	2.010	2.010
η_{21}	14.333	13.75	1.966	1.9693
η_{21}^*	15.333	16.3	2.214	2.1127
η_{22}	22.333	20.25	2.981	2.979
η_{22}^*	21	23.57	3.120	3.097
K_3	15	60	5.250	5.277
K_3^*	61	16.61	5.433	5.325
η_{31}	63	15.88	5.521	—
η_{31}^*	61.333	21.34	5.787	—
η_{32}	63	18.88	5.731	—
η_{32}^*	65	21.66	6.066	—
η_{33}	97	19.13	8.129	—
η_{33}^*	105	30.17	9.462	9.460
K_4	66.6	125	13.412	—
K_4^*	211.27	16.80	15.965	—
η_{41}	194.60	22.25	15.179	—
η_{41}^*	214.60	23.54	16.670	—
η_{42}	194.60	25.25	15.389	—
η_{42}^*	220.60	21.85	16.972	—
η_{43}	194.60	27.38	15.538	—
η_{43}^*	229.27	27.08	17.945	—
η_{44}	257	25.25	19.758	—
η_{44}^*	285	38.27	22.629	—
K_5	130.33	226	24.943	—
K_5^*	459	24.82	33.867	—
η_{51}	397	26	29.610	—
η_{51}^*	465	31.55	34.749	—
η_{52}	397	26	29.610	—
η_{52}^*	473.67	29.87	35.248	—
η_{53}^*	485	35.10	36.407	—
η_{54}^*	499	37.29	37.540	—
η_{55}	534.34	34	39.784	—
K_6	513	371	61.880	—
K_6^*	1136	29.4	81.578	81 ± 1.3 (W)
K_7	239	568	56.490	—
K_7^*	1291.66	~32	92.7	92.4 ± 1.8 (Z)
K_8	262.34	825	76.114	—
K_8^*	1905.01	~35	135.8	—
K_9	2415.40	1150	249.578	—
K_9^*	3537.40	~38	250.3	—
K_{10}	5449	1551	490.00	—
K_{11}	1539.63	2036	1220.074	—

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